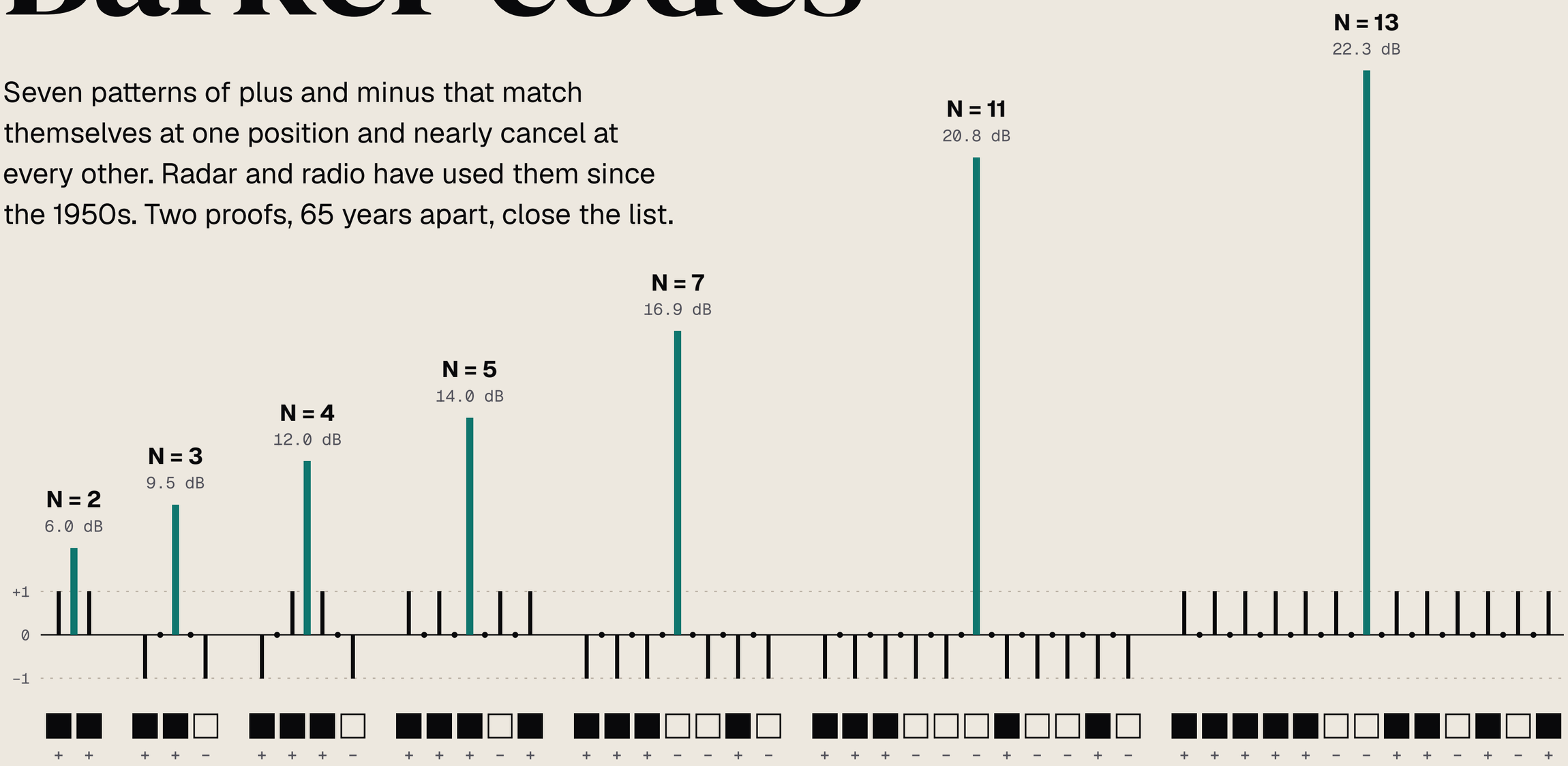
BINARY CODES · AUTOCORRELATION · 1953 TO 2026

The seven Barker codes

Seven patterns of plus and minus that match themselves at one position and nearly cancel at every other. Radar and radio have used them since the 1950s. Two proofs, 65 years apart, close the list.



READING THE FIGURE

Each group of bars compares one code with a shifted copy of itself. The tall bar is the match at zero shift, and its height is the code's length N. Every other bar is −1, 0 or +1.

The squares are the chips of the code, filled for +1 and open for −1. All seven share one vertical scale.

THE PROPERTY

One peak, flat everywhere else

Slide a code along a copy of itself. At each shift, multiply the chips that overlap and add the products. At zero shift every chip agrees, so the sum is the length N.

A Barker code holds every other sum to −1, 0 or +1. No code of plus and minus chips can do better, because an odd number of ±1 terms never adds to zero.

A matched filter sees a peak N times its largest sidelobe. For N = 13 that is 22.3 dB.

THE USE

Sync words, radar pulses, early Wi-Fi

R. H. Barker proposed the idea in 1953 to mark the start of a frame in a stream of bits. A receiver that correlates against the code finds the frame at the peak.

Pulse-compression radar sends the 13-chip code as phase flips inside one long pulse. The echo compresses to one chip wide, so range resolution improves 13 times at the same pulse energy.

In its direct-sequence mode, the first Wi-Fi standard (IEEE 802.11, 1997) spread each symbol over the 11-chip code at 1 and 2 Mbit/s.

THE PROOF

Why there is no eighth

1953 — DEFINED
R. H. Barker asks for a sync pattern whose off-peak autocorrelation stays very low. Codes turn up at lengths 2, 3, 4, 5, 7, 11 and 13, and searches find nothing longer.

1961 — PROOF ON PAPER
R. Turyn and J. Storer prove that no code of odd length exists beyond 13.

2026 — PROOF CHECKED BY MACHINE
A proof produced by an OpenAI model settles the circulant Hadamard conjecture, which rules out every even length beyond 4. A formal version is verified by the Lean proof assistant.

Before 2026, an eighth code was known to need more than 10^{33} chips. Complete proofs had been claimed more than once since 2016. The 2026 paper was released in October and is still being read. The Lean check covers the even lengths only.

CHECKED FOR THIS POSTER

134,217,724 — sequences of plus and minus searched, every one from length 2 to 26

32 — pass the Barker test

7 — codes remain once flipped, reversed and sign-alternated copies are merged

0 — found at any length from 14 to 26

WHY SOME TABLES LIST NINE

Lengths 2 and 4 are often printed twice. The second entry is the first with every other chip flipped, which leaves the size of every sidelobe unchanged. This poster counts each such pair once.

SOURCES

R. H. Barker, Group synchronizing of binary digital systems, in Communication Theory, 1953.

R. Turyn and J. Storer, On binary sequences, Proc. Amer. Math. Soc. 12, 1961.

P. Borwein and M. J. Mossinghoff, Wieferich pairs and Barker sequences, II, LMS J. Comput. Math. 17, 2014.

OpenAI, The circulant Hadamard conjecture, 2026. github.com/openai/math, Apache-2.0 licence.

NOTES

Decibel figures are 20 $\log_{10}$ N, the ratio of the peak to the largest sidelobe.

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VA2TCP with Telos Machina · Sherbrooke, Québec

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